$$(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0.$$

Solving the differential equation, we have

$$y=\frac{c}{2}[x\sqrt{(1-x^2)}+\sin^{-1}x].$$

To verify that this gives the given series, we have by differentiating,

$$\frac{dy}{dx}=c\sqrt{(1-x^2)}=c(1-\frac{1}{2}x^2-\frac{1}{2!\,2^2}x^4-\frac{1.3}{3!\,2^3}x^6-\dots$$

$$-\frac{1.3.5\dots(2n-1)}{(n+1)!\,2^{n+1}}x^{2n+2}-\dots).$$

Integrating, $y=c(x-\frac{1}{2.3}x^3-\frac{1}{2!2^2.5}x^5-\frac{1.3}{3!2^3.7}x^7-\dots$

$$-\frac{1.3.5\dots(2n-1)}{(n+1)!\,2^{n+1}(2n+3)}x^{2n+3}-\dots).$$

Putting $c=-2$, we have

$$\lim_{x\doteq1}[-(x\sqrt{(1-x^2)}+\sin^{-1}x)]=-\frac{\pi}{2}=-2+\frac{1}{3}+\frac{1}{2!\,2.5}+\frac{1.3}{3!2^2.7}+\dots$$

Also solved by G. B. M. Zerr, who found for the limit of the sum of n terms of the series as $n\doteq\infty$, $\frac{5}{3}-\frac{1}{2}\pi$.

GEOMETRY.

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349. Proposed by J. A. CAPARO, Notre Dame University, Notre Dame, Indiana.

Given the radius of a circular smooth cylinder and its position with respect to a source of light and the eye. Find a geometrical construction to determine the line of brilliancy.

Solution by C. N. SCHMALL, New York City.

By a simple examination of the data* it is clear that the problem reduces to the following: Given a circle and two points A, B, outside (in same plane), to find a point P on the circumference, such that AP and

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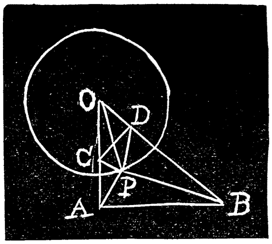
*See *Encyclopedia Britannica*, Vol. XIV, page 589.



BP make equal angles with the radius drawn to P.

This is known as Alhazen's Problem and does not admit of a solution by ruler and compasses only. However, by the use of an hyperbola an approximate solution can be effected.

Construction. Let O be the center of the circle, a its radius. Take C and D so that $AO.OC=a^2=BO.OD$.

Now, the locus of the vertices of the triangles whose base is CD and whose base angles have a constant difference $(OCD-ODC)$ is well known to be a hyperbola. This will cut the circle in four points, of which let P be one. This is the point required.

Proof. $\angle CDP-\angle DCP=\angle OCD-\angle ODC$ (by construction). Transposing and adding,

$\therefore \angle OCP=\angle ODP$...(1).

Also in the triangles AOP, POC, we have $AO:OP=OP:OC$ (by construction).

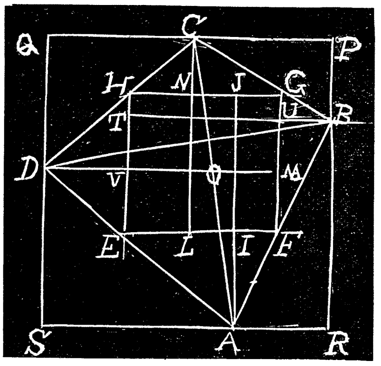
$\therefore \angle APO=\angle OCP$. Similarly, $\angle BPO=\angle ODP$ by (1).

$\therefore \angle APO=\angle BPO$, and $\therefore \angle APR=\angle BPR$. Q. E. F. Q. E. D.

350. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Given the quadrilateral $AB=a=225$, $BC=b=153$, $CD=c=207$, $DA=d=135$, $AC=e=240$. Find the side of the square inscribed in this quadrilateral having a corner in each side.

Solution by the PROPOSER.



Let $ABCD$ be the given quadrilateral, $EFGH$ the inscribed square, $RPQS$ the circumscribed rectangle having its sides parallel to the sides of the square. Draw AIJ, BUT, CNL, DVM perpendicular, respectively, to EF and HG, FG and HE, HG and EF, HE and GF.

Let $AB=a$, $BC=b$, $CD=c$, $DA=d$, $AC=e$, $BD=f$, O the intersection of AC, BD, $\angle COB=\beta$, $\angle ACL=\theta=\angle CAJ$, $\angle BDM=\phi=\angle DBT$, $\angle BAC=\delta$, $\angle DAC=\gamma$, $\angle BCA=\rho$, $\angle DCA=\mu$, area $ABCD=\triangle$.

Then $\phi=\frac{1}{2}\pi-(\beta-\theta)$, $RB=a\cos(\delta-\theta)$, $RA=a\sin(\delta-\theta)$, $DS=d\cos(\gamma+\theta)$, $AS=d\sin(\gamma+\theta)$, $DQ=c\cos(\mu-\theta)$, $CQ=c\sin(\mu-\theta)$, $BP=b\cos(\rho+\theta)$, $PC=b\sin(\rho+\theta)$, $AI+x+CN=AJ+CN=e\cos\theta$, $BU+x+DV=BT+DV=f\cos\phi$.

$\therefore BT+DV=f\sin(\beta-\theta)$.

$x^2+\frac{1}{2}x(AI+BU+CN+DV)=\triangle=\frac{1}{2}x(AI+x+CN)+\frac{1}{2}x(BU+x+DV)$.

$\therefore x[e\cos\theta+f\sin(\beta-\theta)]=2\triangle$...(1).